Distributed power allocation for D2D communications underlaying/overlaying OFDMA cellular networks

Marco Moretti, Andrea Abrardo
Dipartimento di Ingegneria dell’Informazione,
University of Pisa, Italy
Outline

- Introduction
- System model
- Power allocation in D2D networks
  - D2D reuse mode (underlay) mode
  - D2D dedicated (overlay) mode
- Numerical results
- Conclusions
D2D modes

- Device-to-device (D2D) communications, coexisting with infrastructured cellular networks, have the potential of enhancing the total cell throughput, reducing power consumption and increasing the instantaneous data rate.
- In order to provide the system with maximum flexibility, D2D communications are able to operate in multiple modes:
  - Dedicated or overlay mode, when the cellular network allocates a fraction of the available resources for the exclusive use of D2D devices;
  - Reuse or underlay mode, when D2D devices use some of the radio resources together with the UEs of the cellular network.
Resource allocation distributed algorithms

- In both modes power and resource allocation play a vital role for the performance of the D2D system.
- For its nature D2D communications are amenable to a distributed implementation
  - Smaller control overhead
  - Computational load distributed among users
D2D and cellular network

- We consider a scenario where $K$ D2D connections coexist with a cellular network sharing the same bandwidth.

- In *dedicated* mode a fraction of the total available bandwidth is assigned exclusively to D2D transmissions, so that interference between cellular and D2D terminals is completely avoided.

- In *reuse* mode the whole uplink bandwidth is available to each D2D terminal so that D2D nodes and UEs are free to interfere with each other. In this mode the interference from D2D communications to the cellular network must be controlled. This requires a form of centralized control, which actively involves the cellular network.
D2D modes

(a) Dedicated (overlay) mode

(b) Reuse (underlay) mode

Fig. 1. D2D communication modes: (a) Dedicated mode, where D2D connections are assigned a fraction of the total available bandwidth, so that there is no interference between cellular terminals; (b) Reuse mode, where the whole uplink bandwidth is available to each D2D terminal, so that D2D node and UE terminals interfere with each other.

Therefore, employing the Shannon capacity formula, the throughput of the $k$-th couple over the $N$ available links is

$$R_k(p_k) = \sum_{n \in N} \log_2 \left( 1 + \frac{G_{n,k,k} p_{k,n}}{\sum_{j \in K \setminus k} G_{n,j,k} p_{j,n} + \sigma^2_{k,n}} \right),$$

(2)

where $G_{n,k,k} = |H_{n,k,k}|^2$, $p_k = [p_{k,1}, p_{k,2}, \ldots, p_{k,N}]$ is the vector stacking the power transmitted on the $N$ subcarriers by user $k$. The set $P_k = \{p_k \in [0, P_k, 1] \times [0, P_k, 2] \times \cdots \times [0, P_k, N]\}$ is the set of admissible power levels for user $k$ where $P_{k,n}$ is the power mask for user $k$ on subcarrier $n$. 

III. D2D Dedicated Mode: Rate Maximization Under Power Constraint

In dedicated operation mode, the D2D nodes transmit over a fraction of the bandwidth which is dedicated exclusively to D2D transmissions. In this case, we focus on the problem of finding the power allocation that maximizes the sum of the rates of the D2D network with a power constraint per user. Since we are considering a distributed scenario, where each device tries to optimize its performance with a strategy that is influenced by the users' decisions, the presence of interference greatly complicates the problem with respect to the standard waterfilling solution.
D2D and interference

- The interference between D2D connections is managed through distributed power allocation among D2D terminals, with the goal of minimising control from the base station.
- To avoid interference with cellular terminals located in adjacent cells a power mask, i.e., a maximum transmitting power, is imposed to each D2D terminal on each subcarrier.
- Interference from cellular terminals to the D2D receivers is treated as uncontrollable additional noise and is assimilated to thermal noise.
Power allocation in dedicated mode

 Classical rate maximization problem

\[
R(p^*) = \max_{p \in \mathcal{P}} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \log_2 \left( 1 + \frac{G^m_{k,n} p_{k,n}}{\sum_{j \in \mathcal{K} \setminus k} G^m_{j,n} p_{j,n} + \sigma^2_{k,n}} \right)
\]

subject to

\[
\sum_{n \in \mathcal{N}} p_{k,n} \leq P_k \quad k \in \mathcal{K}
\]

\[\mathcal{P} = \mathcal{P}_1 \times \mathcal{P}_2 \times \cdots \times \mathcal{P}_K\] is the set of feasible powers

\[\mathcal{N}\] is the set of subcarriers dedicated to D2D

Not convex (due to interference at the denominator)

We aim at finding a ‘good’ local optimum
Potential games

- A game $G(\mathcal{K}, \{S_k\}_{k \in \mathcal{K}}, \{U_k\}_{k \in \mathcal{K}})$ is a potential game if it exists a potential function $f$ such that for any two arbitrary strategies $x_k, y_k \in S_k$ the following equality holds:

$$U_k(x_k, s_{-k}) - U_k(y_k, s_{-k}) = f(x_k, s_{-k}) - f(y_k, s_{-k}) \quad \forall k \in \mathcal{K}$$

- In potential games best/better response dynamics ALWAYS converge from any arbitrary initial strategy to a Nash Equilibrium, which is also a (local) maximizer of the potential function.
Potential games

- The game $G(K, \{\tilde{P}_k\}, R(p_k, p_{-k}))$, where the players are the $K$ D2D rx-tx couples, the set of strategies for each player $k$ is $\tilde{P}_k = \{p_k \in P_k | \sum_{n \in N} p_{k,n} \leq P_k\}$, the set of all possible power profiles that meet the power constraint $P_k$, and the payoff function is $R(p_k, p_{-k})$, the overall rate maximized with respect to $p_k$ is a potential game and the potential function is the payoff itself.

- The game $G$ is an identical interest game, a game in which the players’ utility functions are chosen to be the same
Potential games

- **Best response dynamics:** each user tries sequentially to maximize its utility function, the power of other users is fixed.

\[
p_k^* = \arg \max_{p_k \in P_k} \sum_{n \in N} \log_2 \left( 1 + \frac{p_{k,n}}{i_{k,n}} \right) \]

\[
+ \sum_{\ell \in K \setminus k} \sum_{n \in N} \log_2 \left( 1 + \frac{p_{\ell,n}}{i_{\ell,k,n} + \frac{G_{\ell,n}}{G_{\ell,\ell}}p_k,n} \right)
\]

subject to

\[
\sum_{n \in N} p_{k,n} \leq P_k \quad k \in K
\]
Potential games

- **Better response dynamic**: each user maximizes
  \[
  \tilde{R}(p_k, p_{-k}; p_k(0)) \approx R(p_k, p_{-k})
  \]
  obtained by linearizing the part b) of the rate \( R(p_k, p_{-k}) \) with its first order Taylor expansion.

- Since the part b) of the rate is convex its linear approximation is an under estimator
Potential games

The linearization yields a new convex problem

\[
\max_{p_k \in P_k} \sum_{n \in N} \log_2 \left( 1 + \frac{p_{k,n}}{i_{k,n}} \right) + \sum_{n \in N} \alpha_{k,n} p_{k,n}
\]

subject to

\[
\sum_{n \in N} p_{k,n} \leq P_k
\]

whose solution is a better response for our problem.

Let \( y_k \) and \( x_k \) be the old power allocation and the solution of the linearized problem, it holds

\[
R(y_k, p_{-k}) = \tilde{R}(y_k, p_{-k}; y_k) \leq \tilde{R}(x_k, p_{-k}; y_k) \leq R(x_k, p_{-k})
\]
The iterative algorithm

- The algorithm may converge to different local optima depending on the scheduling order.
- Given a generic scheduling order the proposed algorithm is implemented as follows:
  1. User $k$ computes the parameters $\alpha_{k,n}$
  2. Solves the linearized problem
  3. Stops if convergence is achieved
- By trying exhaustively all possible scheduling orders we can find the global optimum or a very close approximation of it.
Power allocation for the reuse mode

In this case a new constraint is added to the optimization problem

\[
\max_{p \in \mathcal{P}} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \log_2 \left( 1 + \frac{p_{k,n}}{i_{k,n}} \right)
\]

subject to

\[
\sum_{n \in \mathcal{N}} p_{k,n} \leq P_k \quad k \in \mathcal{K}
\]

\[
\sum_{k \in \mathcal{K}} A_{k,0}^n p_{k,n} \leq Q_n \quad n \in \mathcal{N}
\]

Since we are considering the uplink, a maximum interference level \(Q_n\) is tolerated at the BS.
Upper bound for the reuse mode

- Lagrangian of the allocation problem is

$$\mathcal{L}(p, \nu) = \sum_{k \in K} \sum_{n \in N} \log_2 \left( 1 + \frac{p_{k,n}}{i_{k,n}} \right) + \sum_{n \in N} \nu_n \left( Q_n - \sum_{q \in K} A^n_{q,0} p_{q,n} \right)$$

- A bound for the allocation problem can be found in the dual domain by computing

$$\min_{\nu} g(\nu)$$

subject to

$$\nu \geq 0$$

where the Lagrangian dual function is

$$g(\nu) = \max_{p \in \tilde{P}} \mathcal{L}(p, \nu).$$
Upper bound for the reuse mode

- Computing the Lagrange dual function can be done resorting to the iterative framework developed in the first part.

- Given a value of $\nu$, at each iteration user $k$ has to solve the following problem to find $g(\nu)$

$$\max_{p_k \in \tilde{P}_k} \sum_{n \in N} \log_2 \left(1 + \frac{p_k,n}{i_k,n}\right) + \sum_{n \in N} \alpha_{k,n} p_k,n - \sum_{n \in N} \nu_n A_{k,0}^n p_k,n$$

- The correct value of $\nu$ can be found with the ellipsoid method.
Heuristic for the reuse mode

- From the upper bound we derive an heuristic:
  - We use the local maximizer found with game theory to compute an approximated version of the subgradient of the Lagrangian.
  - The Lagrangian multiplier is updated with the approximated subgradient until convergence
- Converges to a feasible point not necessarily optimum.
Numerical results

Simulation parameters
- Number of cells in the system $B = 1, 3, 7$
- Cell radius 500 m
- Number of subcarriers reserved in overlay mode $N = 8$
- Number of D2D couples $K = 8$ users
- Same $P_{\text{max}} = 0.25$ W for all D2D users
- D2D couples deployed randomly in each cell, with a tx-rx distance uniformly distributed in the interval $[0, D_{\text{max}}]$, with $D_{\text{max}} = 100$ m.
Overlay mode, number of cells $B = 1$

\[ \eta (\text{bit/s/Hz}) \]

- IADRMP-MS
- IADRMP
- SCALE
- IWF
Overlay mode, number of cells $B = 3$
Overlay mode, number of cells $B = 7$

\[ \eta \text{ (bit/s/Hz)} \]

Channel realization index

- IADRMP-MS
- IADRMP
- SCALE
- IWF
Algorithm’s convergence

- Intro
- System model
- Power allocation
- Results
Underlay, number of cells $B = 1$
Underlay, effect of interference constraints

![Diagram showing the relationship between power (P) and efficiency (η) with different interference constraints (Q_max). The diagram illustrates how varying Q_max affects the efficiency at different power levels.](image-url)
Comparison of overlay and underlay modes

- reuse, $Q_{\text{max}} = -132$ (dB), $N_d = 4$
- dedicated, $N_d = 8$
- reuse, $Q_{\text{max}} = -128$ (dB), $N_d = 8$
- dedicated, $N_d = 12$
- reuse, $Q_{\text{max}} = -125$ (dB), $N_d = 12$
Conclusions

- We have shown that the rate maximization power allocation problem can be formulated as a potential game.
- We have proved the convergence of a distributed algorithm for the D2D dedicated mode to a local maximum of the sum rate.
- For D2D reuse mode, the allocation problem is formulated with an additional requirement for each subcarrier so that the total interference at the base station generated by the D2D nodes does not exceed a given threshold.
- After finding the optimal solution, which is too complex for a practical implementation, we have proposed a heuristic algorithm, which builds on the power allocation algorithm devised for the dedicated D2D mode to find a feasible solution.