Degrees of Freedom in Cached Interference Networks with Limited Backhaul

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(A) Motivation
Interference Channels

- No side information between transmitters
  - Weak interference case:
    - Treat interference as noise and focus on detecting the desired signals
  - Strong interference case:
    - Channel orthogonalization (FDMA, TDMA)
    - Interference alignment & interference coordination

- With side information between transmitters
  - Cooperative MIMO
    - Turns interferences into desired signals
    - Improves Spectral Efficiency of Wireless Networks
Favorable vs Unfavorable PHY Topology

- No side information of payload sharing between BSs
  - Unfavorable topology (Interference channel)
- The side information of payload sharing between BSs
  - Favorable topology (MIMO broadcast channel)

No side information of payload sharing
(Interference Channel Topology)

With full side information of payload sharing
(MIMO Broadcast Channel Topology)

3 data streams can be transmitted simultaneously

6 data streams can be transmitted simultaneously

Cooperative MIMO transmission by forming a virtual MIMO broadcast channel using both global CSI and shared payload data
Sharing Payload between Base Stations is the key

How to share data between transmitters (BS)?

- Backhauling
- Expensive Backhaul

High capacity backhaul between BSs is required, which is expensive especially for small cell networks

Induce side information to the BS
- Data requested by the users are in the cache of the BSs
- Induced “side information” is dynamic (depending on the cache state)

Create MIMO Cooperation Opportunity (Cache-state): Induces “Side Information” at the BS

Conventional Caching vs PHY Caching in Wireless Networks

PHY Caching in Wireless Networks

Induce "side information" at the PHY

Dynamic PHY Topology Change of the Radio Interface depending on the cache state

Conventional Caching

Reduce # of hops

Load Balancing

PHY Topology of the Radio Interface is fixed regardless of the cache state

PHY Caching & Conventional Cache are Complimentary
Key Questions

(Q1) What is the DoF Limit for Cache Interference Channel with Limited Backhaul?

(Q2) How to achieve it?
- Design Challenge #1: Cache design to increase the chance of MIMO Cooperation?
- Design Challenge #2: Causality of backhaul?
- Design Challenge #3: Mixed Timescale Adaptation to Content Popularity?

(Q3) Tradeoff between Backhaul and Cache Resource?
- Storage Capacity at Cache and Backhaul Capacity both can contribute to improving the spectral efficiency.
- Which one is a more effective resource?
Opportunistic Cooperation

- No side information (Interference Topology) Current payload requested by the K users are NOT in the PHY-cache.
  - The payload requested by each user can only be served by one BS.

- Full side information (Broadcast Topology) Current payload requested by the K users are in the PHY-cache.
  - The payload requested by users can be served by K BSs using MIMO cooperation.

- Dynamic Side Information (induced by cache state)
  - With global CSI, $\max(\text{DoF})=2$
  - Without CSI, $\max(\text{diversity order})=4$

Cache induced spatial multiplexing gain and spatial diversity gain
Design Challenge #1: Cache Design & MIMO Cooperation Opportunity

- Naïve Caching: Probability of MIMO Cooperation Opportunity exponentially small w.r.t. number of Tx

- System has 4 video files (red / blue / yellow / orange)
- Cache at the BSs store the same content to support Co-MIMO
- Cache capacity limited \( \Rightarrow \) 50% of video files \( \Rightarrow \) Probability that an accessed video file in cache = 0.5
- Co-MIMO Probability = 0.5 x 0.5 (because the video files accessed by each user is independent and asynchronous)
Design Challenge #2: Causality Constraint of Backhaul

**Cache vs Backhaul**
- Side information at the BS can be provided by the cache or backhaul

**Cache-induced Side Information**
- Non-causal because it is pre-fetched at the cache

**Backhaul-induced Side Information**
- Causality Constraint: data from backhaul cannot be used before it is fetched.

**Cache Design and Backhaul Design are coupled**
Design Challenge #3: Mixed Timescale Adaptation to Content Popularity

- **Adaptive Caching to the Content Popularity**
  - Determines the dynamic priority of caching one content over the others based on distribution of requests

- **Adaptive Backhauling to the Content Request Realization**
  - Determines the dynamic priority of fetching one content over the others based on instantaneous requests

- **Mixed Timescale Stochastic Optimization**
  - Complex large-scale stochastic optimization problem
(Q1) DoF Upper Bound of Cached Interference Networks with Limited Backhaul
Cached-Interference Network Model

- K single antenna BSs
- K single antenna users
- Content library consists of L files $X_1, \ldots, X_L$ with equal size $F$
- Each BS is equipped with a **cache** of storage capacity $B_c$, and a **backhaul** with transmission capacity $R$
- User request vector (URV) $\pi = [\pi_1, \pi_2, \ldots, \pi_K]$
System Model

Cache storage allocation vector

\[ \mathbf{q} = [q_{1,1}, \ldots, q_{k,l}, \ldots, q_{K,L}] \]

satisfy the cache storage capacity constraint

\[ q \in \Lambda = \left\{ [q_{1,1}, \ldots, q_{k,l}, \ldots, q_{K,L}] \mid \sum_{l=1}^{L} q_{k,l} F \leq B_{c}, \forall k \in \{1, \ldots, K\}, q_{k,l} \in [0, 1], \forall k \in \{1, \ldots, K\}, \forall l \in \{1, \ldots, L\} \right\}. \]

\( q_{l} \sim \) portion of cache storage allocated to \( l \)-th content

LK mappings \( \{\phi_{k,l}\} \) from content file \( X_{i} \) to the content stores at the cache at BS \( k \) \( W_{k,l} \)

Side information vector realization at the \( t \)-th frame is a random vector \( s(t) \) with empirical distribution \( \Pr[s(t) = \phi^{l}] = p \) with instantaneous data \( v_{k}(t) \) with \( k \) with information vector \( s(t) \) at the \( t \)-th frame.
System Model

K² mappings \( \psi_{k,j}^{m} (\cdot, t) \) from content file \( X_{nj} \) requested by user \( j \) to a subset of \( X_{nj} \) which is fetched from the backhaul of BS \( k \) \( V_{k,1}^{\pi}(t) \)

Backhaul transmission capacity allocation vector

\[
 r^{\pi}(t) = \left[ r_{1,1}^{\pi}(t), \ldots, r_{K,j}^{\pi}(t), \ldots, r_{K,K}^{\pi}(t) \right]
\]

Satisfy the instantaneous backhaul transmission capacity constraint

\[
 r^{\pi}(t) \in \Theta = \left\{ [r_{1,1}, \ldots, r_{k,j}, \ldots, r_{K,K}] \mid \sum_{j=1}^{K} r_{k,j} \leq R, \forall k \in \{1, \ldots, K\}, \right. \\
\left. r_{k,j} \geq 0, \forall k, j \in \{1, \ldots, K\} \right\}
\]

\( r^{\pi}(t) \sim \text{backhaul rate allocated to user } j \text{ at the } k\text{-th BS} \)
System Model

$B_j^\pi(t) \subseteq X_{\pi_j}$ denote the set of bits scheduled for delivery to user $j$ in frame $t$.

$B_j^\pi(t)$ satisfy the user request constraint

$$\bigcup_{t \in [1,T_T]} B_j(t) = X_{\pi_j}, \forall j.$$ and causality constraint

$$B_j^\pi(t) \subseteq \left( \bigcup_{t' \in [1,t]} V_{k,j}(t') \right) \cup W_{k,\pi_j},$$

Side information state

$$s_{k,j}(t) = \begin{cases} 1 & B_j^\pi(t) \neq \emptyset, \\ 0 & \text{otherwise} \end{cases}.$$ Indicates whether the set of bits scheduled for delivery to user $j$, $B_j^\pi(t)$, can be obtained from cache or backhaul at BS $k$.

Side information vector:

$$s(t) = \left[ s_{1,1}(t), \ldots, s_{k,j}(t), \ldots, s_{K,K}(t) \right] \in \{0, 1\}^{K^2}$$
System Model

Cache-Backhaul-Assisted PHY exploits the side information vector $s(t)$ to send the set of $B^\pi(t)$ bits to the $K$ users.
(A) Instantaneous DoF Region with Side Information

- Given a certain side information vector \( s(t) \) at the \( K \) BSs at the \( t \)-th frame, the scheduled bits \( B(t) \) at the \( t \)-th frame is limited by the transmission data rate of the physical layer.

\[
|B_j^\pi(t)| = c_j(H(t), s(t)) \tau
\]

- The rate \( c_j(H(t), s(t)) \) is achievable if the user \( j \) can decode the scheduled bits \( B_j(t) \) successfully at the \( t \)-th frame.

- The set of achievable rate vectors

\[
c(H(t), s(t)) = [c_1(H(t), s(t)), \ldots, c_K(H(t), s(t))]\]

forms the capacity region \( C(H(t), s(t)) \in \mathbb{R}^K \).
Definition 1 (Instantaneous DoF region under given $H(t)$ and $s(t)$): The instantaneous DoF region under given side information vector $s(t)$ and CSI $H(t)$ is defined as [2]

$$
D(H(t), s(t)) = \left\{ (d_1, \ldots, d_K) \in \mathbb{R}_+^K \right\}
$$

$$
\sum_{k=1}^{K} \beta_k d_k \leq \lim_{\Delta \to \infty} \sup_{c \in C(H(t), s(t))} \frac{1}{\log_2 \text{SNR}} \sum_{k=1}^{K} \beta_k c_k
$$

\forall (\beta_1, \ldots, \beta_K) \in \mathbb{R}_+^K

where $\Delta \to \infty$ means that $\text{SNR} \to \infty$, $R \to \infty$, $B_C \to \infty$, $F \to \infty$ such that $\frac{R}{\log_2(\text{SNR})} = \tilde{R}$, $\frac{B_C}{\log_2(\text{SNR})} = \tilde{B}_C$ and $\frac{F}{\log_2(\text{SNR})} = \tilde{F}$.
(A) Instantaneous DoF Region with Side Information

Example of DoF region with side information when $K=2$

$[s_{1,1}=1, s_{1,2}=0, s_{2,1}=1, s_{2,2}=1]$

$[s_{1,1}=1, s_{1,2}=1, s_{2,1}=1, s_{2,2}=1]$
(A) Instantaneous DoF Region with Side Information

Lemma 1 (Outer bound of the instantaneous DoF region $\mathcal{D}(s(t))$ under an instantaneous side information vector $s(t)$ and CSI $H(t)$): $\mathcal{D}(H(t), s(t))$ is outer bounded by

$$\mathcal{D}(H(t), s(t)) \subseteq \hat{\mathcal{D}}(s(t)) = \left\{ [d_1, \ldots, d_K] \left| M_j = \sum_{k=1}^{K} s_{k,j}(t), \right. \right.$$  \hspace{1cm} (12)

$$d_k \leq \min\{M_k, 1\}, \forall k \in \{1, \ldots, K\}, \hspace{1cm} (13)$$

$$d_k + d_j \leq \max\{M_k, 1\}, \forall k \neq j \in \{1, \ldots, K\}, \hspace{1cm} (14)$$

$$\sum_{k:(k,j) \in \mathcal{J}} (M_k - d_k) d_k + \sum_{j:(k,j) \in \mathcal{J}} (1 - d_j) d_j \geq \sum_{(k,j) \in \mathcal{J}} d_k d_j, \forall \mathcal{J} \subseteq \{(k, j) | 1 \leq k \neq j \leq K\}. \hspace{1cm} (15)$$
(B) Empirical Probability Vector of Side Information

The side information vector (induced by cache and backhaul) is given by

\[ s(t) = [s_{1,1}(t), \ldots, s_{k,j}(t), \ldots, s_{K,K}(t)] \in \{0, 1\}^{K^2} \]

There are a total of \(2^{K^2}\) side information patterns. The empirical probability that the \(i\)-th side information pattern occurs is given by

\[ p_\pi(s^{(i)}) = \frac{\sum_{t \in [1,T_\pi]} 1(s(t) = s^{(i)})}{T_\pi}, \]

Denote

\[ p_\pi^{\pi} = \left[ p_\pi^{\pi}(s^{(1)}), \ldots, p_\pi^{\pi}(s^{(2^{K^2})}) \right] \]

as the side information probability vector under URV \( \pi \).

Represents the combined benefit of the cache and backhaul
Flow balance principle:
The amount of side information transmitted from BS $k$ to user $j$ cannot be larger than the amount of information obtained from cache and backhaul at BS $k$ for user $j$.

\[ q_{k,\pi,j} \sum_{s \in S} d_j^\pi (s) p_\pi (s) + \bar{r}_{k,\pi,j} \geq \sum_{s \in S} s_{k,j} d_j^\pi (s) p_\pi (s), \forall j, k. \]  

(18)

\[ \bar{r}_\pi \triangleq \frac{1}{T_\pi} \sum_{t=1}^{T_\pi} r_\pi (t) \] is the average backhaul transmission capacity allocation vector with feasible region $\Theta = \{ \bar{r}_\pi = \frac{1}{T_\pi} \sum_{t=1}^{T_\pi} r_\pi (t), \text{r}_\pi (t) \in \Theta, \forall t \}$

$\bar{r}_\pi (q, \bar{r}_\pi, \{d^\pi (s), \forall s \in S\})$ depends on backhaul capacity allocation only through $\bar{r}_\pi$.

The impact of the backhaul rate allocation policy can be represented by $\bar{r}_\pi$. 
**Upper bound of average sum DoF**

\[
D(K, \tilde{B}_C, \tilde{R}) \leq \max_{\mathbf{q}} \mathbb{E}_\pi \left[ \max_{\mathbf{r}_\pi, \mathbf{d}_\pi(s), p_\pi} \sum_{k=1}^{K} \sum_{s \in S} p_\pi(s) d_k_\pi(s) \right]
\]

\[
\text{s.t. } \mathbf{q} \in \Lambda, \quad (19)
\]

\[
\mathbf{r}_\pi \in \Theta, \quad (20)
\]

\[
\mathbf{p}_\pi \in \Gamma_\pi(\mathbf{q}, \mathbf{r}_\pi, \{\mathbf{d}_\pi(s), \forall s \in S\}), \forall \pi, \quad (22)
\]

\[
\mathbf{d}_\pi(s) \in \mathcal{D}(s), \forall s \in S, \forall \pi, \quad (23)
\]

**Challenge:**

Upper bound is given by complicated non-convex optimization problem, where the objective function is not concave and the constraint (22)-(23) is not convex. Difficult to obtain a closed-form characterization of the upper bound to quantify the impact of the system parameters.
(C) Upper Bound on Average Sum DoF

- **Closed-form average sum DoF upper bound**
  - By relaxing the non-convex constraints and replacing the objective function as a concave function, we obtain a relaxed convex problem, and then obtain a closed-form upper bound.

Theorem 1 (Average sum DoF upper bound with limited backhaul): Consider a cached interference network consisting of $K$ BSs and $K$ users, where the per BS cache storage capacity is $\tilde{B}_C$ and the per BS backhaul capacity is $\tilde{R}$. The $l$-th content file has size $F_l$. A closed-form upper bound of average sum DoF of the cached interference networks over URV distribution $P_{\pi}$ is given by:

\[
D(K, \tilde{B}_C, \tilde{R}) \leq \bar{D}(K, \tilde{B}_C, \tilde{R}) = \min_{\rho_1} \left[ K \sum_{l=1}^{\tilde{B}_C / F} \rho_l + \sqrt{\left( K \sum_{l=1}^{\tilde{B}_C / F} \rho_l \right)^2 + 4K \tilde{R}} \right].
\]

(24)
(C) Upper Bound on Average Sum DoF

Closed-form average sum DoF upper bound

\[ D \left( K, \tilde{B}_C, \tilde{R} \right) \leq \overline{D} \left( K, \tilde{B}_C, \tilde{R} \right) \]

\[ = \min \left[ \frac{K \sum_{l=1}^{\tilde{B}_C/\tilde{F}} \rho_l + \sqrt{\left( K \sum_{l=1}^{\tilde{B}_C/\tilde{F}} \rho_l \right)^2 + 4K \tilde{R}}}{2}, K \right]. \]

- The average sum DoF is limited by both the cache storage capacity \( \tilde{B}_C \) and backhaul capacity \( \tilde{R} \).
- When \( R=0 \) (no backhaul), \( D \sim \min \left\{ K \sum_{l=1}^{\tilde{B}_C/\tilde{F}} \rho_l, K \right\} \). If \( \rho_l = 1/L, \forall l \in \{1, \ldots, L\} \) (equal popularity), then \( D \sim \min \left\{ \frac{K\tilde{B}_C}{\tilde{F}L}, K \right\} \) (consistent with [2]).
- When \( B_C = 0 \) (no cache), then \( D \sim \min \left\{ \sqrt{KR}, K \right\} \).
- In general, if \( \rho_l = 1/L, \forall l \in \{1, \ldots, L\} \) (equal popularity), then

\[ D \sim \min \left\{ \frac{K\tilde{B}_C}{\tilde{F}L} + \sqrt{\left( \frac{K\tilde{B}_C}{\tilde{F}L} \right)^2 + 4K\tilde{R}}}{2}, K \right\}. \]

Both the cache storage capacity and backhaul transmission capacity contribute to average sum DoF.
(Q2) How to Achieve the DoF gain?
Key Design Challenges

• **Design Challenge #1: Improve the Chance of Cooperation**
  - Naive Random Caching will not work

• **Design Challenge #2: Causality Constraint of Backhaul-induced Side Information**
  - Under a naive backhaul scheduling and sequential transmission scheme, the causality constraint cannot always be satisfied.

• **Design Challenge #3: How to allocate the cache storage and backhaul rate w.r.t. different contents?**
  - Complex Mixed Timescale Stochastic Optimization
Achievable Scheme - High Level

- **Key components**
  - Circulant cache content placement
  - Greedy causal backhaul scheduling
  - Hierarchical bit scheduling
    - User group scheduling
    - User subgroup scheduling
    - Round robin bit scheduling in each user subgroup
  - Symmetric constraints to simplify the design

\[
q_{k,l} = q_l, \forall k \in \{1, \ldots, K\}
\]

\[
\bar{r}_{k,j}^{\pi} = \bar{r}_{j}^{\pi}, \forall k \in \{1, \ldots, K\}
\]
Achievable Scheme - Sum DoF

- Achievable sum DoF with given \((q, \{\vec{r}^\pi\})\)

The physical meaning of the variables the key components will be elaborated in the following slides.

An example with the following parameters will be used to illustrate each components of the proposed achievable scheme

**Number of users and BSs** \(K = 3\)

**Size of content library** \(L = 3\)

**Cache storage allocation** \(q = [1, \frac{1}{3}, \frac{1}{6}]\)

**User request vector** \(\pi = [1, 2, 3]\)

**Backhaul capacity allocation** \(\vec{r}^\pi = [0, \frac{1}{6}, \frac{1}{12}]\)
How to adapt Cache Storage and Backhaul Rate?
Adaptive Cache Storage + Backhaul Rate Allocation

- Based on the achievable scheme, we can achieve an average sum DoF of

\[ E_\pi D_B^\pi (q, r^\pi) \]

- We want to maximizing DoF w.r.t. the cache content storage allocation vector \( q \) (adaptive to distribution of URV rather than realization of URV) and the backhaul capacity allocation vector \( r \) (adaptive to the instantaneous realization of URV)

\[
P : \max_{q \in \Lambda} E_\pi \max_{r^\pi \in \Theta} D_B^\pi (q, r^\pi)
\]

Long-term cache storage allocation (Adaptive to distribution of URV)

\[
\sum_{l=1}^{L} q_l F \leq B_C
\]

Short-term cache storage allocation (Adaptive to instantaneous realizations of URV)

\[
\sum_{j=1}^{K} \bar{r}_j \leq R
\]
Adaptive Cache Storage + Backhaul Rate Allocation

- Problem P is a two timescale stochastic optimization problem.
- Furthermore, the objective function is neither convex nor concave.
  - To make the problem trackable, a lower bound of $D_B^\pi(q, \vec{r}^\pi)$ is given by

\[
P' : \max_{q \in \Lambda} \mathbb{E}_{\pi} \max_{\vec{r}^\pi \in \Theta, D^\pi} D^\pi
\]

\[
s.t. D^\pi \leq \min_{j \in \{1, \ldots, K\}} K \left( q_{\pi j} + K \frac{r_{\pi j}}{D^\pi} / D^\pi \right)
\]
Adaptive Cache Storage + Backhaul Rate Allocation

- Optimization decomposition structure

Joint problem $P'$

Sample average approximation

Alternating optimization

Long-time-scale optimization of $q$ for fixed $\{\tilde{r}^{(n)}_\pi\}$

$$\max_{q \in \Lambda, D^{\pi(n)}} \sum_{n=1}^{N} D^{\pi(n)}$$

$$s.t. \frac{1}{K} \left( D^{\pi(n)} \right)^2 - q^{(n)}_\pi D^{\pi(n)} \leq K \tilde{r}^{\pi(n)}_j, \forall j, \forall n$$

Solved by successive linear approximation

Short-time-scale optimization of $\{\tilde{r}^{\pi(n)}_\pi\}$ for fixed $q$

$$\max_{\pi^{(1)} \in \mathbb{N}, D^{\pi(1)}} D^{\pi(1)}$$

$$s.t. D^{\pi(1)} \leq \min_{j \in \{1, \ldots, K\}} K \left( q^{(1)}_\pi j + K \tilde{r}^{\pi(1)}_j / D^{\pi(1)} \right)$$

Solve $r^{(1)}_\pi$ by bisection of $D^{\pi(1)}$

Decoupled to $N$ subproblems

$$\max_{\pi^{(N)} \in \mathbb{N}, D^{\pi(N)}} D^{\pi(N)}$$

$$s.t. D^{\pi(N)} \leq \min_{j \in \{1, \ldots, K\}} K \left( q^{(N)}_\pi j + K \tilde{r}^{\pi(N)}_j / D^{\pi(N)} \right)$$

Solve $r^{\pi(N)}_\pi$ by bisection of $D^{\pi(N)}$
Adaptive Cache Storage + Backhaul Rate Allocation

<table>
<thead>
<tr>
<th></th>
<th>Low complexity cache storage and backhaul transmission capacity allocation algorithm</th>
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<tbody>
<tr>
<td><strong>Performance</strong></td>
<td>The algorithm converges to a <strong>stationary point</strong> of problem $P'$ with probability 1.</td>
</tr>
<tr>
<td><strong>URV requirements</strong></td>
<td><strong>Does not require</strong> knowledge of the URV distribution.</td>
</tr>
<tr>
<td><strong>Complexity</strong></td>
<td><strong>Short-term backhaul transmission capacity allocation:</strong> $O(\log(K/\sigma))$, where $\sigma$ is the accuracy of bisection.</td>
</tr>
<tr>
<td></td>
<td><strong>Long-term cache capacity allocation:</strong> Scale with $N$, which is the number of samples used in the sample average approximation</td>
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Optimality of the Achievable scheme

- The proposed achievable scheme is DoF optimal in the following cases:
  - **High skewness content popularity (Zipf $\gamma \to \infty$):**
    - The user requests concentrate on a few content files.
    - The optimal cache storage allocation is to cache the most popular files.
  - **Low skewness content popularity (Zipf $\gamma \to 0$):**
    - The content popularity distribution is flat.
    - The optimal cache storage allocation is uniform caching (i.e., $q_i = \frac{P_C}{P_D}, \forall i$)
  - **Cache storage capacity is zero:**
    - The user requests are served by the backhaul, which is adaptive to the realization of URV.
- The achievable scheme also has good performance in general cases, which will be illustrated by numerical results.
(Q3) Tradeoff between Cache Storage and Backhaul Capacity
Tradeoff between Cache Storage and Backhaul Transmission Capacity

- **Cache Storage vs Backhaul Transmission Resource**
  - Increasing the cache capacity at BSs is a much more cost-effective way to enhance the DoF performance than increasing the backhaul capacity.

**Corollary 1 (Cache Storage vs Backhaul Transmission Resource):** For uniform content popularity (i.e., $\rho_l = 1/L$, $\forall l \in \{1, \ldots, L\}$) and sufficiently large $K$, the average sum DoF upper bound

$$
D(K, \tilde{B}_C, \tilde{R}) = \begin{cases} 
K\tilde{B}_C, & \tilde{R} = 0, \\
\frac{F_L}{\sqrt{K\tilde{R}}}, & \tilde{B}_C = 0.
\end{cases}
$$

- **Increase $\tilde{B}_C$ by $N$ times**
  - $\rightarrow$ increase the average sum DoF by $N$ times

- **Increase $\tilde{R}$ by $N^2$ times**
  - $\rightarrow$ increase the average sum DoF by $N$ times

The average sum DoF at PHY is proportional to the amount of side information available at all the BSs. If the side information comes from the BS cache, then we only need to increase the cache storage capacity by $N$ times. If the side information comes from the backhaul, then the backhaul capacity needs to increase by $N^2$ because the duration of the communication session is also reduced by $N$ times.
Tradeoff between Cache Storage and Backhaul Transmission Capacity

Corollary 2 (Tradeoff between cache storage and backhaul capacity)

• Amount of backhaul resource that can be saved by adding a unit cache storage.
  • A larger \( \Delta B_c \) means that a unit cache storage can trade for more backhaul capacity.

\[
\Delta \beta_c \Delta B_c = \rho_l
\]

Critical (\( \gamma = 1 \))
Sub-critical (\( \gamma = 0.3 \))
  The amount of the backhaul capacity trade from unit cache storage is non-zero, even if the increased cache capacity is much smaller than the total content size.

Super-critical (\( \gamma = 1.2 \))
  The amount of the backhaul capacity trade from unit cache storage is non-zero, even if the increased cache capacity is much smaller than the total content size.

(c) Tradeoff curve under Zipf popularity under different \( \gamma \) with \( L = 1000 \) and \( D = 5 \).

\[ \frac{\tilde{R}}{B_c / (FL)} \]
Numerical Performance
System Performance

Simulation setup
- A cached MIMO interference network consists of and $K = 10$ BSs and $K=10$ user.
- The size of the content library $L = 200$.
- The popularity of content files follow the Zipf distribution with skewness $\gamma$.

Baselines
- **Baseline 1**: Average sum DoF upper bound of cached interference network.
- **Baseline 2**: Proposed achievable scheme + cache most popular files ($q_i = \begin{cases} 1 & l = 1, \ldots, \tilde{B}_C / \tilde{F} \\ 0 & l > \tilde{B}_C / \tilde{F} \end{cases}$) + equal backhaul allocation ($\tilde{F}_j^\pi = R / K$)
- **Baseline 3**: Proposed achievable scheme + uniform caching ($q_i = \frac{\tilde{B}_C}{\tilde{F} L}, \forall l$) + equal backhaul allocation ($\tilde{F}_j^\pi = R / K$)
- **Baseline 4**: DoF of $K$-pair interference network without cache
System Performance

- DoF versus cache storage capacity with $\gamma = 1.5, R=1$

Baseline 1: Average sum DoF upper bound

Baseline 2: Proposed achievable scheme + cache most popular files + equal backhaul allocation

Baseline 3: Proposed achievable scheme + uniform caching + equal backhaul allocation

Baseline 4: DoF of tradition K-pair interference network

Proposed achievable scheme + optimized cache and backhaul capacity allocation:
Close to the DoF upper bound, outperforms other baselines
System Performance

- **DoF versus backhaul transmission capacity with** $\gamma = 1.5, \, B_c/L_F = 3/20$

Baseline 1: Average sum DoF upper bound

Baseline 2: Proposed achievable scheme + cache most popular files + equal backhaul allocation

Baseline 3: Proposed achievable scheme + uniform caching + equal backhaul allocation

Baseline 4: DoF of tradition K-pair interference network

Proposed achievable scheme + optimized cache and backhaul capacity allocation:
Close to the DoF upper bound, and outperforms other baselines
System Performance

- **DoF versus content popularity skewness** with $R = 1$, $Bc/LF = 3/20$

![Graph showing system performance with different baselines.]

- **Baseline 1:** Average sum DoF upper bound
- **Baseline 2:** Proposed achievable scheme + cache most popular files + equal backhaul allocation
- **Baseline 3:** Proposed achievable scheme + uniform caching + equal backhaul allocation
- **Baseline 4:** DoF of traditional K-pair interference network

Proposed achievable scheme + optimized cache and backhaul capacity allocation:
- Close to the DoF upper bound
- For low skewness, uniform caching is optimal
- For high skewness, cache most popular files is optimal
(F) Conclusions
Conclusions

- Upper bound of DoF in Cached Interference Channel with Limited Backhaul

- Achievable Scheme - DoF optimal in some cases.
  - Flexible Scheduling of Bits at PHY is critical to enhance the probability of cooperation
  - Autonomous Learning of Caching and Backhaul Priority w.r.t Content Popularity

- Tradeoff between Cache Storage and Backhaul Capacity
  - Cache Storage is more efficient to enhance the PHY DoF
References


